Verb-based Curriculum for Mathematics Education

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• Knowledge-based society require more than knowledge.
• From Quantity of knowledge to Quality of knowledge (knowledge about knowledge, systematization of knowledge)
• Mathematics plays an important role in this quality of knowledge.
Mathematical activity

• Counting, measuring are examples of mathematical activities. They are primary activity, and are reflected and deepened.
Activity deepens itself.

How can we express “deepening activity” in the curriculum?

Pirie and Kieran (1994)
Content

• 1. Initial idea of verb-based curriculum
• 2. Characteristics of activity and verb
• 3. Analysis of Japanese Course of Study in terms of verbs
• 4. Structure of verb-based curriculum
1. Initial idea of verb-based curriculum
Background

• New course of study put emphasis on learning mathematics “through mathematical activity” at all stages of school education (MEXT 2008).
• According to constructivism (Nakahara 1995), children play a major role in mathematical activity not only on the real objects but also on mathematical objects.
Six universal mathematical activities

• From different perspective, Bishop (1991) argues that each culture has developed mathematics and there exist *six universal activities* to be observed at the basement of these mathematics.

• How to count -> grouping, base-ten, addition

• How to represent -> place value system
## Initial idea of verb-based curriculum

<table>
<thead>
<tr>
<th></th>
<th>Noun-based curriculum</th>
<th>Verb-based curriculum</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Focus</strong></td>
<td>Product</td>
<td>Process</td>
</tr>
<tr>
<td></td>
<td>the definition, concept, and formula to enable the speed and accuracy of solution</td>
<td>mathematical thinking and ideas</td>
</tr>
<tr>
<td><strong>Structure</strong></td>
<td>Noun</td>
<td>Verb</td>
</tr>
<tr>
<td></td>
<td>Arrangement and structure of knowledge</td>
<td>Arrangement and structure of activity</td>
</tr>
</tbody>
</table>

Baba, Iwasaki (2001)

the essence of activity is expressed in terms of verbs.
the objectives of this paper

• (1) To clarify characteristics of verbs,
• (2) To analyze the course of study in terms of verbs, and
• (3) To propose an example of verb-based curriculum.
2. Characteristics of activities and verbs
More nouns than verbs

- Nouns occupy higher percentage than verbs among basic vocabulary, and the percentage goes even higher as the number of basic vocabulary in Japanese language.

The graph of the percentage of nouns and verbs in basic vocabulary
Instantaneousness of activity

- The reason why verbs as signifier are fewer than nouns, lies in the property of activity as signified that is instantaneous and does not retain its locus of movement very long. For example, the activity 'to count' can be perceived by means of eyes and ears, but it only remains as an afterimage for a while and then its existence cannot be perceived by our senses any longer.
Elasticity of verbs

• The way of counting sheep and the way of counting sheets of paper are different. Probably the former is pointing at each sheep at a time, and continues one after another until it covers all. The latter may be turning over the pages.
3. Analysis of Course of Study in terms of verbs

A: Numbers and Calculations
B: Quantity and Measurements
C: Geometrical Figures
D: Mathematical Relations

Section 3 Arithmetic

1. OVERALL OBJECTIVES
Through mathematical activities, to help pupils acquire basic and fundamental knowledge and skills regarding numbers, quantities and geometrical figures, to foster their ability to think and express with good perspectives and logically on matters of everyday life, to help pupils find pleasure in mathematical activities and appreciate the value of mathematical approaches, and to foster an attitude to willingly make use of mathematics in their daily lives as well as in their learning.

II. OBJECTIVES AND CONTENT FOR EACH GRADE
[Grade 1]
1. Objectives

(1) Through activities using concrete objects and so on, to help pupils enrich their number sense. To help them understand the meaning and the representations of numbers, and to help them understand addition and subtraction, and explore ways of the calculations, and use the calculations.

(2) Through activities using concrete objects and so on, to help pupils enrich their experiences that will form the foundation for understanding quantities and measurements, and enrich their sense of quantities.

(3) Through activities using concrete objects and so on, to help pupils enrich their experiences that will form the foundation for understanding geometrical figures, and enrich their sense of geometrical figures.

(4) Through activities using concrete objects and so on, to help pupils represent numbers and quantities as well as their relations by using words, numbers, algebraic expressions, figures and diagrams and interpret such representations.

2. Content
A. Numbers and Calculations
(1) Through activities such as counting the numbers of concrete objects, to help pupils understand the meaning of numbers and use numbers.
   a. To compare numbers of objects by making one-to-one
How to analyze the Course of Study

(1) Collect all verbs and quasi-verbs from the course of study (MEXT 2008) in Japanese
   • Here quasi-verb is a noun in Japanese that behaves like a verb when it is suffixed by ‘suru’, which means ‘to do’. For example, measurement (測定) is suffixed by suru (する) and becomes do measurement, which is equivalent of measuring activity.

(2) Translate them into English using the English version of the course of study (MEXT 2008).

(3) Analyze them according to grades and domains
Grade 1 Content

A. Numbers and Calculations

(1) Through activities such as counting the numbers of concrete objects, to help pupils understand the meaning of numbers and use numbers.
   a. To compare numbers of objects by making one-to-one correspondence between objects.
   b. To correctly count or represent the number and order of objects.
   c. To make a sequence of numbers and to put numbers on a number line by judging the size and the order of the numbers.
   d. To consider a number in relation to other numbers by regarding it as a sum or difference of other numbers.
   e. To understand the representations of two-digit numbers. ...
<table>
<thead>
<tr>
<th>Verb</th>
<th>Grade 1</th>
<th>Quasi-verb</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>count(数える), understand(理解する), use(用いる), through(通す), correspondence(対応する), represent, put...(on the number line)(表す)</td>
<td>compare(比べる), judge, explore(考える), make(作る), regard, consider(みる), in relation to(関係づける), get to know(知る)</td>
</tr>
<tr>
<td>B</td>
<td>compare (比較する), enrich(豊かにする), through(通す)</td>
<td>form the foundation (基礎となる), read(読む), compare(比べる)</td>
</tr>
<tr>
<td>C</td>
<td>familiar (身の回りにある) through(通す), recognize(認める), grasp(とらえる), use(用いる)</td>
<td>concerning(に関する), express(言い表す), enrich(豊かにする)</td>
</tr>
<tr>
<td>D</td>
<td>use(用いる), represent(表す)</td>
<td>interpret(読み取る)</td>
</tr>
</tbody>
</table>
Basic results

• 66 verbs were collected in all. They consist of 60 transitive and 6 intransitive verbs. Transitive verbs are paid more attention to, because this research is to consider children’s mathematical activities on objects, which can be expressed as transitive verbs. For example, children “measure the weight”.

• Some of them are no longer verbs (e.g. through (通す), nor correspond with one English word “judge” and “explore” for Japanese “考える”.

Further analysis

Making activity an object of another activity

The objects of the activity may be objects found in the environment and mathematical activity itself. The latter is called an “objectification” of activity. There are three types of such objectification such as quasi-verbs, how to, and masu-form. For example, in “explore ways of the calculation (G2 計算の仕方を考える)”, “understand the meaning of measurements (G2 測定の意味を理解する)”, and “deepen their understanding of the representations of integers (G3 整数の表し方についての理解を深める).
Further analysis

The role of verb ‘to deepen’ and ‘to extend’
The verbs, ‘to deepen’ and ‘to extend’, similarly represent the nature of mathematical activity. When new knowledge has been developed based activity towards the previous knowledge that has been learned before. For example, “deepen their understanding of the meaning and the representations of numbers” in the grade 2 assumes the learning and understanding of the numbers in the grade 1.
Example: “objectification” of activity

• (G1) enrich their number sense, using concrete objects and understand the meaning and the representations (how to represent) of numbers ->

• (G2) enrich their number sense, using concrete objects and deepen their understanding of the meaning and the representations (how to represent) of numbers
4. Structure of verb-based curriculum

- Count -> represent -> understand
- Representation (How to represent) -> consider
G1 using **concrete objects**, enrich their number sense. understand the meaning and the representations of **numbers**, 
Through activities such as counting the numbers of **concrete objects**, to help pupils understand the meaning of **numbers** and use **numbers**. 
To compare numbers of **objects** by making one-to-one correspondence between **objects**. 
To correctly count or represent the number and order of **objects**. 
To make a sequence of numbers and to put numbers on a number line by judging the size and the order of the numbers. 
To consider a number in relation to other numbers by regarding it as a sum or difference of other numbers. 
To understand the representations of **two-digit numbers**. 
To get to know the representations of **three-digit numbers** in simple cases. 
To consider numbers using ten as a unit.

G2 using **concrete objects** enrich their number sense. deepen their understanding of the meaning and the representations of **numbers**, 
To help pupils understand the meaning and the representations of numbers, and extend their ability to use numbers. 
To count objects by arranging them into groups of the same size, or by classifying them. 
Up to **four-digit numbers**, to understand the representations of numbers, understand size and order of numbers by the decimal positional numeration system. 
To understand relative size of numbers by regarding 10 or 100 as a unit. 
To consider a number in relation to other numbers by regarding it as a product of other numbers. 
To get to know **simple fractions** such as 1/2 and 1/4.
<table>
<thead>
<tr>
<th>Count</th>
<th>Concrete objects (1)</th>
<th>Up to four digits (1, 2)</th>
<th>Man digits, integers (3)</th>
<th>Fractions, decimals (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 to 1 correspondence</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Compare, Make a sequence of numbers</td>
<td></td>
<td>Arrange them into groups</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Represent number and order of objects</td>
<td></td>
<td>Up to four-digit numbers</td>
<td>ten-thousands</td>
<td>hundred million and trillion</td>
</tr>
<tr>
<td>Consider in relation to</td>
<td></td>
<td>relative size by 10 or 100 as a unit</td>
<td>10 or 100 times or 1/10 of</td>
<td></td>
</tr>
<tr>
<td>Understand</td>
<td></td>
<td>representations of numbers, decimal positional numeration system</td>
<td>Deepen understanding of relative size of numbers</td>
<td>summarize the decimal positional numeration system</td>
</tr>
</tbody>
</table>
### Addition and Subtraction of Natural Numbers

<table>
<thead>
<tr>
<th>Know</th>
<th>Situations are used</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Understand</td>
<td>Meaning based on 1-digit, algorithm, column forms</td>
<td>based on 2-digit numbers, using algorithms in column forms</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Use, extend</td>
<td>Calculation</td>
<td>Calculation</td>
<td>Calculation accurately &amp; appropriately</td>
<td>Use of calculation</td>
</tr>
<tr>
<td>Explore</td>
<td>Ways of calculation</td>
<td>Properties</td>
<td>Properties</td>
<td></td>
</tr>
<tr>
<td>Make use</td>
<td>Properties</td>
<td>Properties</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Check</td>
<td>The results</td>
<td>the results</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consolidate</td>
<td></td>
<td></td>
<td>Calculate</td>
<td></td>
</tr>
</tbody>
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Reference