

The challenges for teachers of helping students acquire a range of mental strategies to solve problems

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Presentation to SEAMEO delegation, 2
December 2011

Background

- Numeracy Development Projects (NDP) designed to raise students' achievement in mathematics and build teachers' professional capacity to teach mathematics.

Key features of NDP

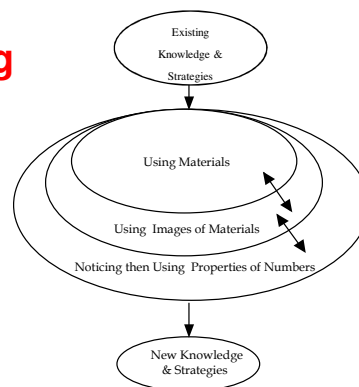
- Number Framework**
 - Strategy (Additive, Multiplicative, Proportional Domains)
 - Knowledge (Sequence, Basic Facts, Place Value)
- Individual Diagnostic Interview**
 - Mental strategies
- Teaching Model**
 - Materials-> imaging-> abstraction, number properties

Professional Learning and Development

- Whole school & at least one year

The Teaching Model

(NDP Book 3 p. 5)



Number Framework (Additive Domain)

- 0-1 Emergent & One-to-one counting (can't add)
- 2-3 Counting All to Add (with materials or by imaging)
- 4 Counting On or Back to Add or Subtract**
- 5 Early Additive Part-Whole Thinking
 - Splitting and recombining numbers to add or subtract
- 6 Advanced Additive Part-Whole Thinking
 - Choosing from a flexible range of strategies to add or subtract whole numbers
- 7 Advanced Multiplicative Part-Whole
 - Choosing from a flexible range of strategies to add or subtract fractional or decimal quantities

Percentages of students in Yr 1-8 at or above mathematics standards

Year	1	2	3	4	5	6	7	8
Maths Standard	Stages 2-3	Stage 4	Early Stage 5	Late Stage 5	Early Stage 6	Late Stage 6	Early Stage 7	Late Stage 7
No. of students	24931	27947	29720	30576	31475	32526	27286	27998
Stage 2+	87	96	99	99	99	100	99	99
Stage 3+	41	76	92	97	98	99	98	99
Stage 4+	19	57	84	94	97	98	98	98
Stage 5+	2	14	41	63	75	84	86	90
Stage 6+		1	5	14	25	38	46	58
No. of students	23106	27376	29955	31021	31946	33051	28149	28445
Stage 7+			1	3	9	19	25	36

Choosing from a range of mental strategies available for Addition & Subtraction problems

- Standard Place-Value Partitioning
- Bridging through Ten/Tidy Number (Rounding)
- Knowledge of Doubles Facts
- Rounding & Compensating
- Balancing (Addition only)
- Reversibility (Subtraction only)
- Equal Additions (Same Difference) (Subtraction only)

Part-Whole Strategies for Addition & Subtraction

Strategy	Addition (eg, 17+15)	Subtraction (eg, 33-17)
Standard Place Value Partitioning	$17+15 = (10+7) + (10+5) = (10+10) + (7+5) = 20+12 = 32$	$30+3-(20+7)=(30-10) + (3-7) = 20 + -4 = 16$ so $33-17=16$
Bridging through Ten/Tidy Number	$17+3 = 20, 20+10=30, 30+2=32$, so $17+15=32$	$33-3=30, 30-10=20, 20-4=16$, so $33-17=16$
Knowledge of Doubles Facts	$17+15=(15+2)+15=(15+15)+2=30+2=32$	$33-17=(16+16+1)-17=16+(16+1)-17=16$ so $33-17=16$
Rounding & Compensating	$20+15=35, 35-3=32$, so $17+15=32$	$33-20=13, 13+3=16$, so $33-17=16$
Balancing	$17+15=(17+3)+(15-3)=20+12=32$	Not Applicable to Subtraction
Reversibility	Not Applicable to Addition	$17+?=33, 17+(3+10+3)=33$, so $17+16=33$, so $33-17=16$
Equal Additions	Not Applicable to Addition	$(33+3)-(17+3)=36-20=16$, so $33-17=16$

Problem Structure: Word Problem Types

- **Join** (Change: Add To)

$$5 + 8 = \square, 5 + \square = 13, \square + 8 = 13$$

- **Separate** (Change: Take From)

$$13 - 5 = \square, 13 - \square = 8, \square - 5 = 8$$

- **Combine** (Part-Part-Whole)

$$5 + 8 = \square, 13 - 5 = \square$$

- **Compare** (Difference)

$$13 - 5 = \square, 5 + 8 = \square, 13 - 5 = \square$$

(Adapted from Carpenter et al., 1999 and Fuson, 1992)

Separate versus Compare

Separate (Change: Take from) **Compare** (Difference)

- *Result Unknown*
 - Ana had 13 plums. She gave 5 to Sam. How many plums did Ana have left?
 - $13 - 5 = \square$
 - *One* quantity that is operated on
- *Difference Unknown*
 - Ana has 13 plums. Sam has 5 plums. How many more plums does Ana have than Sam?
 - $13 - 5 = \square$
 - *Two* quantities that are compared

Partitioning Strategies help students learn about Number Properties

Associative Property (Grouping)

When *adding*, the way the numbers are *grouped* does not affect the answer - the answer remains the same.

NB: Not applicable to *subtraction*

Partitioning & Number Properties contd.

Commutative Property (Order)

Numbers can be *added* in any *order*. The representation may be different, but the answer remains the same.

NB: Not applicable to *Subtraction*

Number Properties for Addition & Subtraction contd.

Inverse Relationships (*Reverse*)

Subtraction can be *reversed* by adding an amount to the addend

Supporting the Additive Thinking of Students

Purpose of the Study

- To investigate what actually happens in classrooms as teachers use NDP resource materials to build students' repertoire of part-whole strategies for solving addition and subtraction problems.

Participants

- 9 Teachers (7 females, 2 males) working at Year 5-6 level in 4 schools (decile 2-10)
- 2-25 yrs teaching experience, 2-7 yrs NDP experience
- Teachers chose an instructional group they judged ready for the lessons (~stage 6)
- 64 children aged 8 to 11 years old

Teachers involved in the research

Teacher	School Decile	Yrs Teaching	Yrs NDP	Group size
Ann	2	25	6	8
Ben	2	9	6	8
Cara	3	10	6	10
Dot	3	6	5	8
Ed	10	14	2	5
Fay	10	20	6	6
Gail	9	2	2	5
Hana	9	13	7	7
Iris	9	6	5	7

Method for gathering data

- Pretest before first lesson (paper and pencil)
- Audiotaped teacher's language
- Observed and recorded nonverbal data
- Interviewed students and teachers after the lesson for highlights and reflections
- Post-test after third lesson (paper and pencil)
- The lessons (NDP Book 5)
 - Saving Hundreds
 - Equal Additions**
 - A Balancing Act

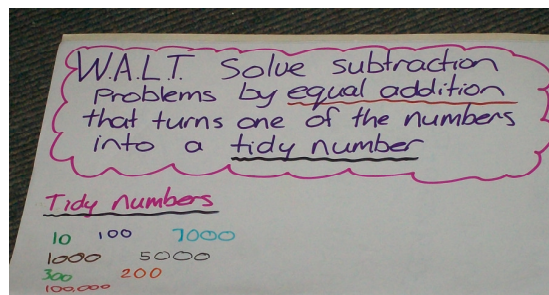
The Problem

“Debbie has \$445 in her bank account, and her younger sister Christine has \$398. How much more money does Debbie have?”

The Problem

- “Debbie has \$445 in her bank account, and her younger sister Christine has \$398. How much more money does Debbie have?”
- Make piles of \$445 and \$398. “Now suppose that Grandma gives Christine \$2 to give her a ‘tidy’ amount of money. To be fair, Grandma gives Debbie \$2 also.” Discuss why $445 - 398$ has the same answer as $447 - 400$ and then record $445 - 398 = 47$ on the board or modelling book.

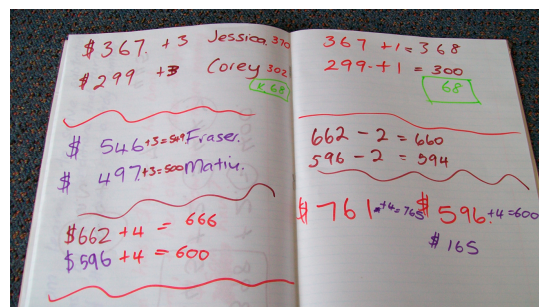
We Are Learning To (WALT)....



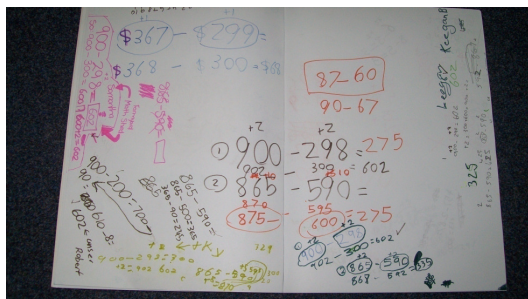
Group recording book and money used with the lesson



Rounding up the Minuend or the Subtrahend?



The group work book



Effective Teaching Practices

- Personalizing the problem for the students (by putting their names into the word problems).
- Fairness (to justify the need for Equal Additions).
- Focusing on ideas of **Difference & Comparison** (instead of on making tidy numbers)
- Proving that the difference remained constant (by building up smaller quantity to equal larger quantity, using number line and pegs).
- Consistency of language and mathematical structure (by understanding **Separate** versus **Compare** problems).

Frequency of “Indicative” words and use of Equal Additions strategy on Post-test

Teacher	“difference”	“how much more”	“why”	“how”	Group size	Children using Equal Additions
Ann	0	1	6	20	8	
Ben	9	3	8	39	8	B4 (B5)
Cara	12	7	7	22	10	C3, C4, C7 (C9)
Dot	0	0	32	29	8	D8
Ed	4	4	7	31	5	E4
Fay	1	3	3	47	6	F3
Gail	32	6	23	31	5	G1, G2 (G5)
Hana	2	4	14	48	7	H1, H4
Iris	0	2	6	15	7	I5

Effective Teaching Practices *contd.*

- The importance of recording the process clearly
- Recording **steps separately**
eg, NOT $298 + 2 = 300 + 60 = 360$
INSTEAD $298 + 2 = 300$, $300 + 60 = 360$
- Ensure **maximum of 9** for any place-value denomination (eg, \$1, \$10, \$100)
- Ensure students always refer to **total value** (not face value) eg, 900-298 -> 902-300 “*nine hundred minus three hundred*” not “*nine minus two then add two zeros*”

More Findings

- Teachers took a calculational instead of a conceptual approach to teaching subtraction.
- Students seemed to be very instrumental rather than being relational in their understanding.
- Teachers stuck closely to NDP lesson (prescriptive), rather than adapting and modifying it (illustrative).
- Teachers kept high level of control over who could talk, about what, and when. Students should be encouraged to take more initiative and responsibility.

Conclusions

Teachers need:

- a deep and connected understanding of the underlying **structure** of the mathematics in order to teach students effectively.
- deeper Pedagogical Content Knowledge (**PCK**) in order to anticipate the likely difficulties students may experience in understanding the Additive domain.

Recommendations

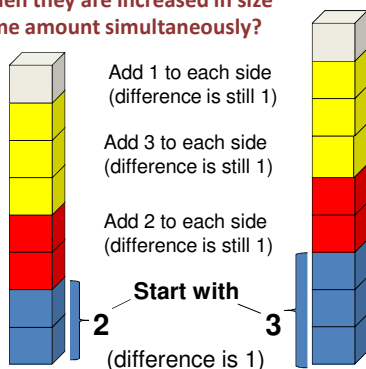
- Start with students’ preferred strategies and **link** to Equal Additions strategy, encouraging students to debate the efficiency of strategies.
- Involve the students in **questioning** and **justifying** their use of strategies to each other.
- Use blocks and small numbers to **illustrate** the idea of adding equal amounts to two collections.
- Use structured Number Lines to show how the difference is **constant** when equal amounts are added to both the minuend and subtrahend.
- Ask the students to do **recording** of processes.

We would suggest it is important to:

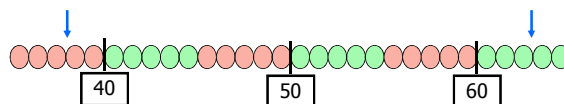
Try introducing the idea using concrete materials that may show the ‘difference’ more clearly, using single-digit quantities initially.

e.g. Unifix Cubes

What happens to the difference in height between 2 towers when they are increased in size by the same amount simultaneously?



Equal differences



What is the difference between 63 and 38?

What is the difference between 65 and 40?

Recommendations contd.

Teachers need to:

- understand Problem Structure, different problem types, and use a variety.
- be aware of the various strategies for solving problems and ensure that students understand these.
- understand how number properties are linked to various strategies.

Finally...

- Teaching Addition & Subtraction is far more complicated than we often assume.

nzmaths website

- <http://www.nzmaths.co.nz/numeracy-projects>
- [Information about the NDP](#)
- [Books](#): PDFs of the NDP books and information on ordering [hard copies](#).
- [Material masters](#): used in NDP activities.
- [Equipment animations](#): show how to use common classroom equipment to teach numeracy concepts.

Importance of **Subitizing** (seeing at a glance) to develop ideas of cardinality and part-whole (Find the Odd One Out)

