
Mathematics Exploration Problems

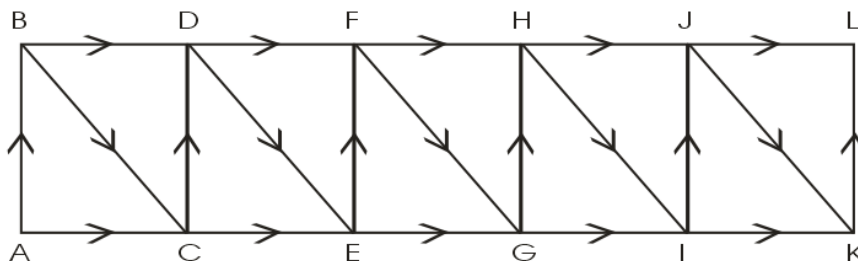
International
Mathematics and Science Olympiad (IMSO)
for Primary School 2005

Jakarta, November 13-19, 2005

Instructions:

- * Write down your name and country on answer sheet.
- * Answer all 6 questions.
- * You have 120 minutes to work on this test.
- * Write down your answer on the provided answer sheets.
- * Use pen to write your answer.
- * Use pencil only to draw figures.

1. The following figure shows a road map of the Gotthem City. Every road is one way, as indicated by the arrow.



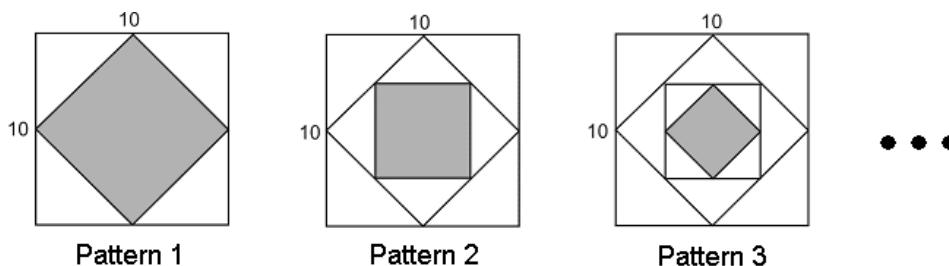
Questions:

- (a) [1 point] How many possible routes are there from A to D?
 - (b) [2 points] How many possible routes are there from A to H?
 - (c) [3 points] How many possible routes are there from A to L?
2. There are some people playing a card game. On the table there are fifty cards, numbered 1 to 50, all facing up. Each player is allowed to choose a certain number of cards. If the sum of the numbers on all the cards chosen by the player is the highest, then he/she is the winner. There is only one winner.

Questions:

What is the lowest possible score that makes a player a sure winner if each player has to choose:

- (a) [2 points] two cards?
 - (b) [2 points] three cards?
 - (c) [2 points] five cards?
3. Consider the following sequence of figures.



The sides of large squares of all patterns are the same, that is 10 cm. The vertices of the shaded squares are the midpoints of the respective side of the outer square.

Questions:

- (a) [3 points] Determine the area of each shaded region in Pattern 1, Pattern 2, and Pattern 3.
- (b) [3 points] If the pattern continues, determine the area of the shaded region in Pattern 10.

4. We can put together four equilateral triangles with 1 unit side length to form many shapes (on the plane) observing the following conditions:
- (i) The shape must be connected.
 - (ii) No triangles overlap.
 - (iii) A triangle must be connected to the rest of the shape by at least a side or part of a side.

For example, shape in Figure A satisfies all the conditions. Its perimeter is 6 units. Shapes in Figure B, Figure C, and Figure D are *not* allowed, but shapes in Figure E, Figure F, and Figure G are allowed.

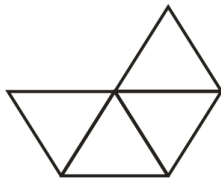


Figure A

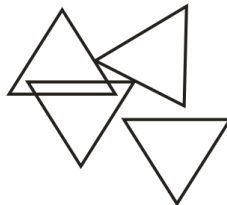


Figure B

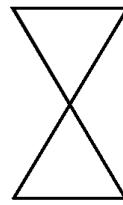


Figure C

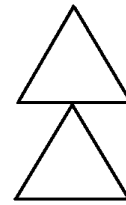


Figure D

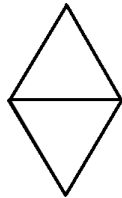


Figure E

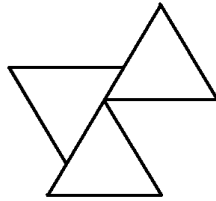


Figure F

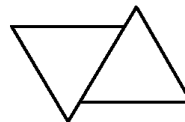
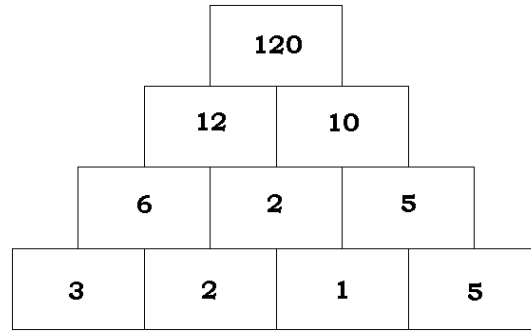


Figure G

Questions:

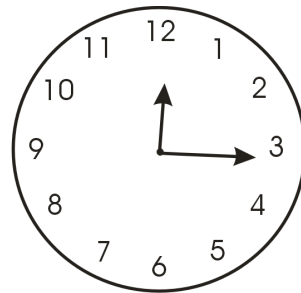
- (a) [1 point] Using four triangles, draw one shape with perimeter 7 units.
- (b) [2 points] Can you make a shape with perimeter 11 units using four triangles? If yes, draw one example.
- (c) [1 point] What is the smallest whole number that can be the perimeter of a shape made with six triangles?
- (d) [2 points] What is the largest whole number that can be the perimeter of a shape made with six triangles?

5. Ten rectangles form a pyramid. Each rectangle is filled in with a positive whole number following a certain rule. In the figure below, the numbers 3, 2, 1 and 5 are placed at the bottom rectangles, thereby resulting to 120 at the top rectangle of the pyramid.



Questions:

- (a) [2 points] What is the rule of filling in the numbers into the rectangles of this pyramid?
- (b) [1 point] Using the rule in (a), what number should be at the top rectangle of the pyramid if you put 1, 2, 1 and 3 (in this order) at the bottom rectangles?
- (c) [3 points] By following the same rule, the number 2160 is obtained at the top rectangle of the pyramid. Find all possibilities of the four numbers placed at the bottom rectangles of the pyramid, if the number 1 is not used? Note: The sequence 3, 2, 2, 4 is considered different from the sequence 4, 2, 2, 3.
6. The hands of the clock shown below form a right angle. The estimated time, to the nearest minute, is 12:16.



Questions:

- (a) [$1\frac{1}{2}$ points] When will the clock hands form a right angle for the first time after 12:16? Estimate the time to the nearest minute.
- (b) [$1\frac{1}{2}$ points] After 1 o'clock, estimate to the nearest minute, when the clock hands will form a right angle for the first time.
- (c) [3 points] From 2:00 a.m. to 12 noon of the same day, how many times will the clock hands form a right angle?