WHAT RESEARCH SAYS ABOUT MATHEMATICAL PROBLEM SOLVING

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The development of the ability to solve problems has long been recognized as one of of the major goals of mathematics education. Every individual in our society is faced with making decisions, they must have the ability to think creatively, laterally, divergently, rationally, objectively, and systematically. Teaching mathematical problem-solving means teaching how to: define the problem to be solved, devise a plan, choose appropriate strategies, collect and analyze relevant information, evaluate relevant information, evaluate the results and make decisions.

A problem was defined by Cooney, Davis & Henderson (1975) as: "… a question which presents a challenge that *cannot* be resolved by some *routine* procedure known to students." Two types of problems have been identified (Charles, 1982: le Blanc, Proudfit & Putt, 1980): (1) standard textbook (translation) problems, and (2) process problems. In solving *translation problem*, the emphasis is on translating a real word situation in the problem into mathematical terminology or mathematical sentence in the solution. The Translation problem requires only the application of skills, principles, or concepts known to students, while the *process problem* requires, in addition, the use of strategy or some non-algorithmic approach. Process problems emphasize the process of obtaining the solution rather than solution itself.

Solving problems is one of the most difficult activities in the mathematics curriculum at all grade levels. The *National Assessment of Educational Progress (NAEP)* reported in 1988 that only 29 percent of large national sample of 17-years-old in the USA were able to solve the following problem:

> Lemonade cost 95c for one 56 once bottle. At the school fair, Bob sold cups holding 8 ounces for 20c each. How much money did the school make on each bottle?

In 1980, the National Council of Teachers of Mathematics (NCTM) recommended that problem-solving should be the focus of the school mathematics in the United States. Although problem-solving is one of the major goals of mathematics education many students still have difficulties with this important task. The performance of United States' students increased from NAEP II to NAEP IV. However, in reporting the fourth NAEP results Kouba et al (1988) stated: "Students have trouble with items that do not involve routine, familiar tasks."

WHAT IS NEEDED IN SOLVING PROBLEMS

Shoenfeld (1985), in his book Mathematical Problem Solving, described four requirements for solving mathematical problems:

1. *Resources*. Mathematical knowledge possessed by the individual that can be brought to bear on the problem at hand.
2. *Heuristic*. Strategies and techniques for making progress on unfamiliar or nonstandard problems; rules of thumb for effective problem solving
3. *Control*. Global decisions regarding the selection and implementation of resources and strategies.
4. *Belief Systems*. One's mathematical world view", the set of (not necessarily conscious) determinants of an individual's behavior.

Problem solving activities in school focus mostly on instructional techniques such as problem-solving strategies, Polya's four steps method (understanding, the problem, devising a plan, carrying out, and looking back), and the teaching of computer programming languages such as LOGO or BASIC (Frank, 1988). This means that research on the teaching of problem-solving has been concerned largely with heuristics, rather than with other requirements such as students' beliefs system.

Students' beliefs, views, ideas, and conceptions of mathematics are developed in the classroom over a long period. Inevitably, students' beliefs about mathematics can help or hinder them as good problem solvers. (Garafalo, 1987; Erlwanger, 1975). Research on students' beliefs about mathematics has revealed: mathematics to be regarded as computation (Frank, 1988), rule based (NAEP IV), and mostly memorizing (NAEP IV); formal mathematics has little or nothing to do with real thinking (Schoenfeld, 1985); the primary aim in mathematics is to get the answer (Confrey, 1980; Frank, 1988); and mathematics problems are solved in less than 10 minutes (Schoenfeld, 1985) and in a few steps (Frank, 1988).

As mathematics teachers, one of our tasks is to help students to develop an awareness of their cognitive functioning, so that they are better able to control and regulate their cognitive actions during their problem solving activities. Teaching strategies are required to focus students' attention on their assumptions and beliefs. For example, teacher question such as: "Why did you use this strategy?"; "Are you sure about this pattern?"; "What happens if x is a negative number?", or "Why do you think you usually make this error?" can help students to become more aware of their cognitive functioning as a first step towards evaluating and modifying it.

Recent research has attempted to find ways of better shaping students' beliefs about mathematics. Frank (1988) suggested four strategies for mathematics teachers to help their students develop positive beliefs about mathematical problem-solving activities:

1. *Start problem-solving early*
2. *Be sure your problems are problems, i.e., non-routine*
3. *Focus on solution, not answer*
4. *de-*emphasize computation

IMPLICATION FOR MATHEMATICS EDUCATION IN INDONESIA

The Indonesian Department of Education and Culture (Depdikbud, 1987) formulated the following aims of mathematics teaching for primary and secondary school students:

(a) to enable learners to be able to successfully tackle situations in their changeable lives, through action training based on thinking; logically and rationally, critically and accurately, objectively, creatively, and efectively
(b) to enable learners to apply their knowledge of mathematics correctly in their daily lives and in other subjects

The first aim can be achieved if students learn how to solve process problems which require logical, rational, creative, and systematic thinking, and ingenuity in conception and reflection. The second aim can be attained if students also learn how to solve translation problems which emphasize translating real-world situations into mathematical terminology and solving the problem by using mathematical principles or mathematical concepts.

Since 1982, problem-solving has been discussed during the in-service and On-service training courses for secondary school mathematics teachers. In 1987, the team of Indonesian instructors of mathematics provided a collection of problems appropriate for secondary school students. However, the intended curriculum, which is prescribed in the national syllabuses, must be completed on time. This forces teachers to focus on the products, or learning outcomes rather than on processes such as problem-solving. This is compounded by mathematical instruction which is focused too much on content and not enough on mathematical behavior.

We need to change this situation. Indonesian students must be active learners rather than mere knower of mathematical fact and procedures. Mathematics teachers in Indonesia should committed to their primary mission to help learners to be better problem-solvers. This commitment is based on our mathematics teaching aims. Based on the research findings described above we should be aware that mathematical problem-solving instruction should not focus on only on resources and heuristics, but also on students' belief system.

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